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Nonlinear concrete model for an internally confined hollow reinforced concrete column

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A hollow reinforced concrete (RC) column has been used to obtain a more economical design by reduction of the material or the self weight of the column. Although a hollow RC column offers a high effectiveness of section properties, it may show poor ductility owing to brittle failure at its inner face. In order to develop a more economical and safer model for a RC column, an internally confined hollow (ICH) RC column and its non-linear concrete model were proposed with the considerations of confining effects. To determine the relation of stress–strain for the concrete in an ICH RC column, possible failure modes of the column were investigated and the confining pressure was derived from equilibriums for each failure mode. A computer program was developed with respect to the derived equilibrium equations and parametric studies were performed to evaluate the application of the developed program. The ICH RC column significantly improved its strength and ductility by the internal confinement.

Introduction

A hollow reinforced concrete (RC) column has been popularly used to obtain a more economical design by reducing its material or self weight. Also, the hollow RC column offers a high effectiveness of section properties but it may show poor ductility owing to brittle failure at the inner face of the column. The brittle failure results from the absence of confinement at the inner face of the hollow RC column where the concrete at the outer face is appropriately confined by transverse reinforcements.

Figure 1 shows a difference in the stress–strain relations between confined and unconfined concrete. Confined concrete shows larger strength and ductility than unconfined concrete. This enhanced strength and ductility of confined concrete depends on the confining pressure. The confining pressure can be controlled by the use of outer transverse reinforcements in a general RC column. In a hollow RC column, however, the confining pressure may not be controlled by only the outer transverse reinforcements because there is no confining pressure at the inner face of the column. Figs 2 and 3 show a hollow RC column without internal transverse reinforcements and with internal transverse reinforcements, respectively. The concrete wall element in Fig. 2 is biaxially confined by axial stress (f_1) and circumferential stress (f_2). In this case, a brittle failure originates from the inner face of the column. The concrete wall element in Fig. 3 is triaxially confined by

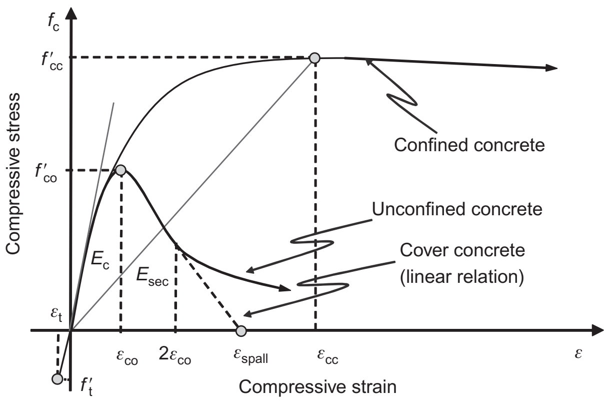

Fig. 1. Confined and unconfined stress–strain relations for monotonic compression loading; reprinted from Popovics[1] with permission from Elsevier

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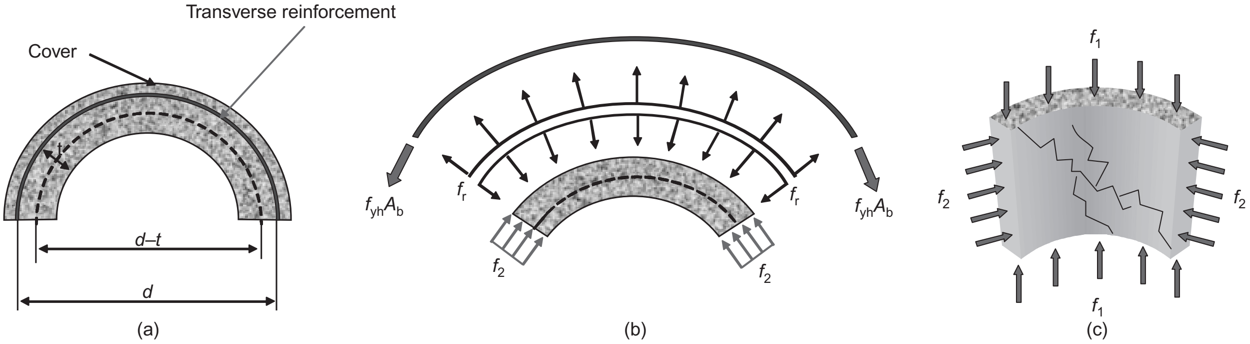

Fig. 2. Hollow RC column without internal hoop reinforcement (biaxially confined state)

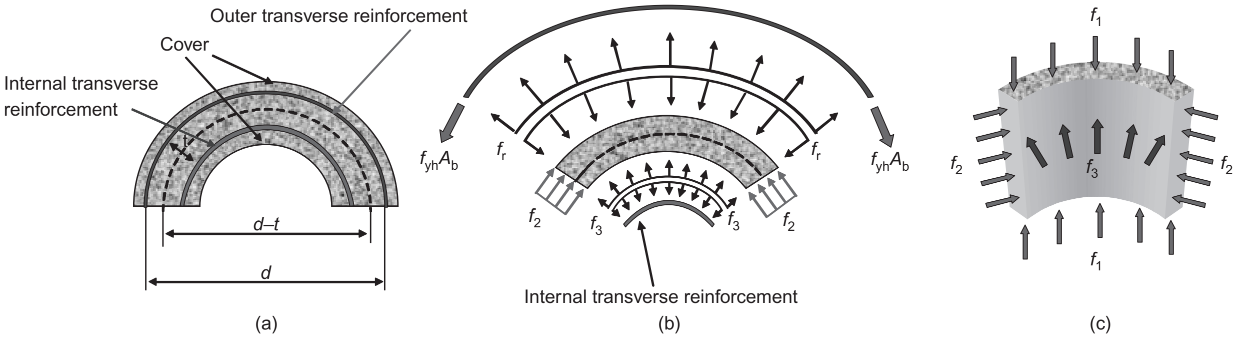

Fig. 3. Hollow RC column with internal hoop reinforcement (triaxially confined state)

axial stress (f_1), circumferential stress (f_2) and radial stress (f_3). The internal transverse reinforcements offer confining stress in the radial direction. The hollow RC column with internal transverse reinforcements produces more strength and ductility than the hollow RC column without internal transverse reinforcements.[2–6]

Many studies have been performed on the confining effect of concrete in a RC column. The result of experimental tests for rectangular transverse reinforcements conducted by Roy and Sozen[2] demonstrated that there was no increase in concrete strength, but a significant increase in ductility could be achieved. Iyengar *et al*.[3] and Desayi *et al*.[6] showed that there was a significant increase in both strength and ductility of concrete members including circular steel spirals. Vallenas *et al*.,[7] Sheikh and Uzumeri[8] and Scott *et al*.[9] tested more realistically scaled specimens of real building columns and presented the stress–strain relations of the specimens. The test results for confined square sections with rectangular and octagonal shaped transverse reinforcements indicated that the strength and the ductility of the specimens were significantly enhanced. However, the empirical formulations for the stress–strain relations of the specimens were widely different to define the descending stress–strain path. Recently, analytical models for the explanation of the effect of concrete confinement were made.[10–14] The analytical results showed widely divergent opinions about the increase in strength and ductility of confined concrete as the shape of the confined section. Mander *et al*.[14] suggested a unified concrete model to define the stress–strain relations for square and circular sections. It is widely used to explain the effect of concrete confinement. Furthermore, the model can define the stress–strain relations of the confined concrete with one function while the models define ascending and descending stress–strain paths with different functions.

Although a hollow RC column shows enhanced performance when it has internal transverse as shown in Fig. 3, these internal transverse reinforcements cannot offer continuous confinement because the reinforcements are not located continuously and they can be easily buckled after spalling of internal cover concrete. Fig. 4 shows that the behaviour of the hollow RC column with the buckled inner transverse reinforcement is similar to that of the hollow RC column without the inner transverse reinforcement of Fig. 2. Thus, a brittle failure of the column may occur after the buckling of

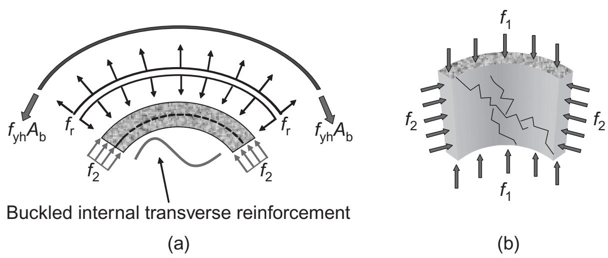

Fig. 4. Biaxially confined state of concrete after buckling of an internal hoop reinforcement

the internal transverse reinforcement. To prevent this brittle failure, it is necessary to offer strong and durable confinement. Fig. 5 shows a hollow RC column with an internal steel tube. The strength and ductility of the column are totally enhanced because of the continuous confining stress provided by the internal steel tube. The current paper therefore investigates a non-linear concrete model of an internally confined hollow (ICH) RC column based on the study of Mander *et al*.[14,15] Fig. 6 shows a typical model investigated in this study. Failure modes of an ICH RC column were defined considering yielding and buckling conditions of the internal tube. An analytical model of the concrete in an ICH RC column was developed considering the confining effect. A brief result from a numerical parameter study was provided using a developed analytical program.

Development of analysis model

For the moment–curvature analyses under combined axial and flexural load, it is necessary to use constitutive models that accurately trace the stress–strain path of the material used. In this research, a new stress–stain model for ICH RC column is developed. This study was performed on the basis of the following basic assumptions

(*a*) the internal tube offers complete confining pressure unless it fails
(*b*) the internal tube offers no confining pressure if it fails
(*c*) the RC member is ruptured when a transverse reinforcement fails.

Uniaxial stress–strain model

Mander *et al*.[14] proposed a unified stress–strain approach to predict the pre- and post-yield behaviour of confined concrete members subjected to axial compressive stress. In this approach, Mander *et al*.[14] proposed concrete models for a monotonic compressive and tensile loading condition, a cyclic compressive and tensile loading condition and cyclic reloading branches. One of these models is briefly reviewed to develop a new model for ICH RC column.

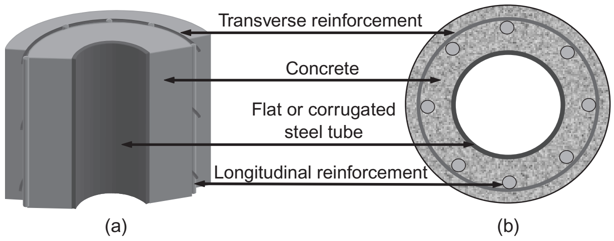

Fig. 6. Cross section of a circular ICH RC column

Figure 1 shows the form of the stress–strain relation in monotonic compression for confined and unconfined concrete suggested by Popovics.[1] Mander *et al*. used equation (1) proposed by Popovics to develop the unified stress–strain relation of the confined concrete subjected to the monotonic compression

$$f_c = \frac{f'_{cc}xr}{r-1+x^r} \quad (1)$$

where $x = \frac{\varepsilon}{\varepsilon_{cc}}$, $r = \frac{E_c}{(E_c - E_{sec})}$, $E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}}$, f_c is concrete stress, f'_{cc} is confined strength of concrete, ε is uniaxial strain, ε_{cc} is strain at peak concrete strength and E_c is the tangent modulus of unconfined concrete.

The tangent modulus of unconfined concrete (E_c) can be estimated as $500\sqrt{f'_{cc}}$ (MPa). The peak concrete strength for confined concrete (f'_{cc}) and the strain at peak concrete strength (ε_{cc}) can be calculated by equations (2) and (3), respectively. The strain at peak concrete strength is given as a function of the strain at peak unconfined strength of concrete (ε_{co}). The value of ε_{co} is usually regarded as 0·002

$$f'_{cc} = f'_c\left(2{\cdot}254\sqrt{1+\frac{7{\cdot}94f'_l}{f'_c}} - \frac{2f'_l}{f'_c} - 1{\cdot}254\right) \quad (2)$$

$$\varepsilon_{cc} = \varepsilon_{co}\left[1+5\left(\frac{f'_{cc}}{f'_c}-1\right)\right] \quad (3)$$

where f'_c is unconfined strength of concrete, f'_l is

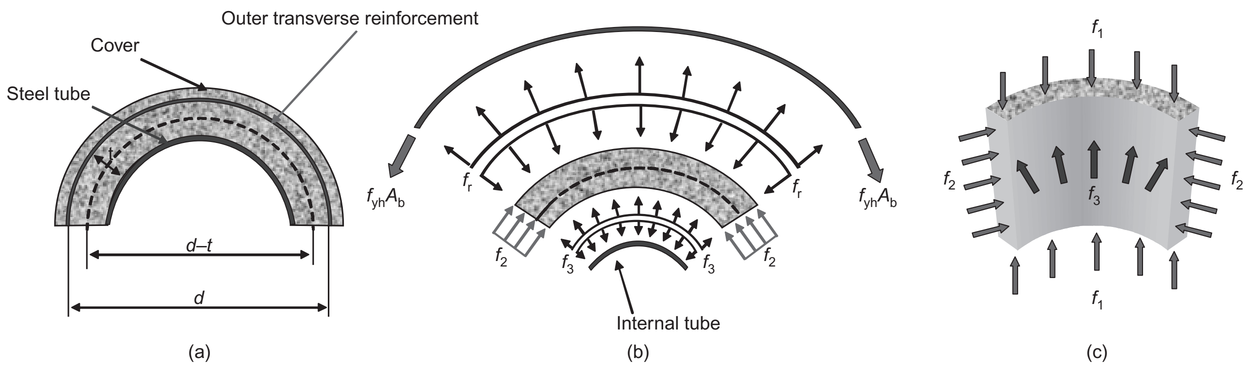

Fig. 5. Triaxially confined state of concrete in ICH RC column by an internal tube

effective constant confining pressure and ε_{co} is strain at peak unconfined strength of concrete.

Figure 7 shows the half section of the confined concrete. For a circular concrete section, the constant confining pressure (f_l) can be derived from the Fig. 7. If the uniform hoop tensile stress (f_s) developed in the transverse reinforcement exerts a lateral pressure on the core concrete, the constant confining pressure can be calculated by equation (4)

$$f_l = \frac{2 f_s A_b}{s d_s} \quad (4)$$

where s is the spacing between adjacent two transverse reinforcements and A_b is the cross-sectional area of transverse reinforcement.

Transverse reinforcements cannot confine the entire core concrete because of the spacing between transverse reinforcements. The effective constant confining pressure (f'_l) therefore has to be used in equation (2) instead of the constant confining pressure (f_l) of equation (4). The effective constant confining pressure can be calculated by equation (5) below using the reduction coefficient (k_e)

$$f'_l = k_e f_l \quad (5)$$

Equilibriums in a hollow RC column

A hollow RC column is not triaxially confined because of the absence of core concrete to develop the confining stress in the radial direction. Thus the concrete in a hollow RC column is assumed to be in the state of biaxial confinement as shown in Fig. 8. The transverse reinforcement in Fig. 8 will not exert any lateral pressure on the core concrete in the radial direction because the core concrete is no longer in the confining state. There are only circumferential pressures on the concrete by arching actions. The confining stress in the circumferential direction (f_{lc}) can be calculated by the same method as the general solid RC member as equation (6). The confining stress in the radial direction can, however, be assumed to be zero $f_{lr} = 0$. From the assumption of $f_{lr} = 0$ and $f_{lc} \neq 0$, the concrete in a hollow RC member can be considered to be in the state of biaxial confinement, and the effective lateral pressure (f'_l) at yield can be defined as the average of radial and circumferential confining stress components f_{lr} and f_{lc} as in equation (7)

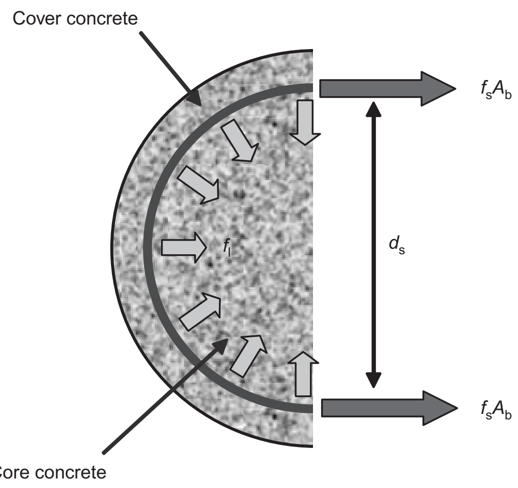

Fig. 7. Confinement forces on concrete from circular hoop reinforcement (Mander et al.[14]), with permission from ASCE

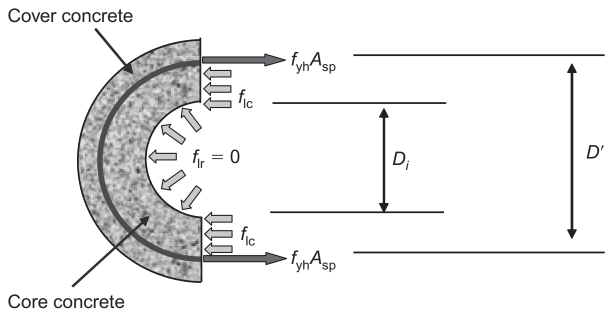

Fig. 8. Confining stress on concrete in a hollow RC column

$$f_{lc}(D' - D_i)s = 2 f_{yh} A_{sp} \quad f_{lc} = \frac{2 f_{yh} A_{sp}}{(D' - D_i)s} \quad (6)$$

$$f'_l = 0{\cdot}5(f'_{lr} + f'_{lc}) \quad (7)$$

where $f'_{lc} = 0{\cdot}95 f_{lc}$ (Mander *et al.*[15]), D' is the diameter of confined concrete, D_i is the diameter of hollow section, f_{yh} is the yield strength of a transverse reinforcement, and A_{sp} is the sectional area of a transverse reinforcement.

Mander *et al.*[15] researched the multiaxial strength criterion when concrete is confined by different confining pressures along each direction. Based on the results of the experimental study, the chart shown in Fig. 9 was suggested according to the ratio of the confining stress and confined strength. It includes graphs of biaxial and triaxial confining conditions. By regression of a curve for the biaxial condition in the chart, the confined strength of concrete in a hollow RC column can be defined as equation (8)

$$f'_{cc} = -2{\cdot}75 \frac{f'^2_l}{f'_c} + 1{\cdot}835 f'_l + f'_c \quad (8)$$

Equilibriums in internally confined hollow RC (ICH RC) column

Before the failure of the inner steel tube or the yield of the outer transverse reinforcement occurs, the core concrete of the ICH RC column is completely subjected to triaxial stresses. Fig. 10 shows the free body diagram of the ICH RC column. Considering the failure of the internal steel tube and the yielding of the outer transverse reinforcement, three failure modes for the ICH RC column can be defined as equation (9). In the first failure mode, the inner steel tube is under buckling or yielding before the yield of the outer transverse

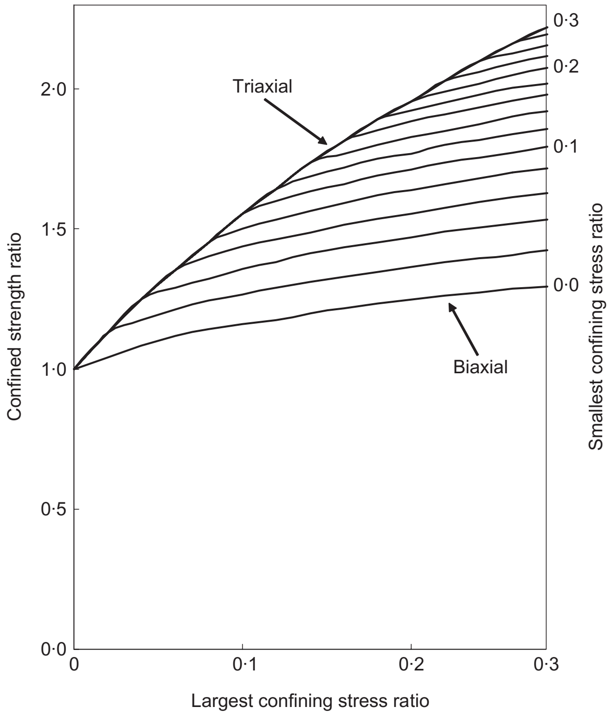

Fig. 9. Confined strength and confining stresses (Mander et al.[14]), with permission from ASCE

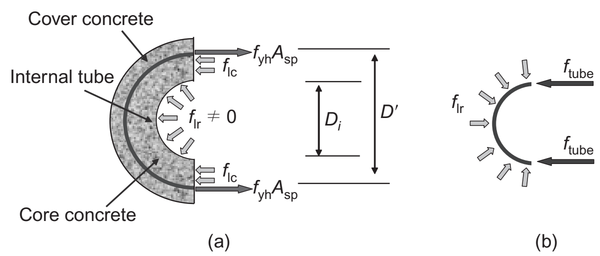

Fig. 10. Confining stress on concrete in an ICH RC column

reinforcement occurs. The reverse is the condition of the second failure mode. In the third failure mode, the failure of the inner steel tube and the outer transverse reinforcement occur simultaneously.

$$f_{tube} > f_{lim} = \text{smaller}(f_{yt}, f_{bk}) \quad \text{Failure mode 1} \quad (9a)$$

$$f_{tube} < f_{lim} = \text{smaller}(f_{yt}, f_{bk}) \quad \text{Failure mode 2} \quad (9b)$$

$$f_{tube} = f_{lim} = \text{smaller}(f_{yt}, f_{bk}) \quad \text{Failure mode 3} \quad (9c)$$

where f_{tube} is compressive stress acting on the internal steel tube in the radial direction (i.e. centripetal stress), f_{yt} is yield strength of the internal steel tube, f_{bk} is buckling strength of the internal steel tube and f_{lim} is the smaller value between yield strength and buckling strength of internal tube.

In the first failure mode, the core concrete of ICH RC columns is in a triaxially confining state before the failure of the inner steel tube occurs. However, after the failure of the internal steel tube, the internal steel tube can be assumed to exert no more confining pressure. Therefore, the core concrete in this state can be assumed to be in a biaxially confining state. In the second failure mode, the core concrete is completely confined until the whole member fails by the yielding of an outer transverse reinforcement. This is because the internal steel tube can exert confining pressure only before the yielding of the outer transverse reinforcement. The failure of the whole member depends on the yielding of the outer transverse reinforcement. Because the triaxial confinement is completely maintained until the failure of the outer transverse reinforcement, the concrete of this member behaves similarly to that of a solid RC column. The confining pressure in the circumferential direction is equal to that of the radial direction ($f_{lc} = f_{lr}$). The third failure mode is a rare case and it has a similar failure pattern to that of the second failure mode. The equilibrium equation for failure modes 2 and 3 can be obtained using equation (10), see also Fig. 10(a). In equation (10), the circumferential stress (f_{tube}) of the inner steel tube can be calculated by equation (11) using Fig. 10(b). Substituting equation (11) in to equation (10), the equilibrium equation can be rewritten as equation (12). Therefore, the relation of the confining pressure (f_l) and the yield strength of the outer transverse reinforcement (f_{yh}) can be obtained as equation (13)

$$[f_l(D' - D_i) + 2f_{tube}t]s = 2f_{yh}A_{sp} \quad (10)$$

$$f_{tube} = \frac{f_l D_i}{2t} \quad (11)$$

$$\left[f_l(D' - D_i) + 2\frac{f_l D_i}{2t}t\right]s = 2f_{yh}A_{sp} \quad (12)$$

$$f_l = \frac{2f_{yh}A_{sp}}{D's} \quad (13)$$

where f_l is the confining pressure of concrete ($= f_{lc} = f_{lr}$), t is thickness of the internal steel tube.

Yield condition of internal steel tube

As the yield strength and the buckling strength of a steel tube depend mainly on the thickness of the tube, the failure mode can be controlled by the thickness of the internal steel tube. Considering the yielding condition of the internal tube, equation (14) is given from equation (11). From equations (11) and (13), the relation of the stress of the inner steel tube and the yield strength of the outer transverse reinforcement can be obtained as equation (14). To derive the failure of the transverse reinforcement before the internal steel tube yields, the stress acting on the internal tube should be smaller than the yield strength of the internal tube. From the stress requirement of the inner steel tube presented in equation (15), the thickness requirement can be calculated by equation (16). If the thickness of the internal steel tube satisfies equation (16), it does not yield until the outer transverse reinforcement yields

$$f_1 = \frac{2t}{D_i} f_{\text{tube}} = \frac{2 f_{\text{yh}} A_{\text{sp}}}{D' s} \tag{14}$$

$$f_{\text{tube}} = \frac{D_i f_{\text{yh}} A_{\text{sp}}}{D' s t} < f_{\text{yt}} \tag{15}$$

$$t > \frac{D_i f_{\text{yh}} A_{\text{sp}}}{D' s f_{\text{yt}}} = t_{\text{yt}} \tag{16}$$

where t_{yt} is the required minimal thickness of tube to avoid its yielding.

Buckling condition of flat internal tube

The internal steel tube of an ICH RC column is unilaterally restrained by concrete. Owing to this unilateral boundary condition, the internal steel tube has different buckling strength from that of an arch or a ring with bilateral boundary conditions. The buckling strength of unilaterally restrained arches has been studied by many researchers.[16–19]

Kerr and Soifer[16] proposed the buckling strength of a circular arch by linearisation of the prebuckling deformation. Haftka *et al.*[17] suggested the bifurcation buckling strength and snap-through buckling strength of a circular shallow arch using the Koiter's method. When a circular shallow arch is buckled in snap-through behaviour, its deformed shape is similar to the buckled shape of a unilaterally restrained arch. Table 1 presents the buckling load coefficients for circular arch suggested by Kerr and Soifer[16] and Haftka *et al.*[17] Sun and Natori[18] proposed the numerical solution of a large deformation problem considering the unilateral boundary condition. Smith *et al.*[19] and Bradford *et al.*[20] showed that the unilateral buckling strength is larger than the bilateral buckling strength for a rectangular plate subjected to the bending, compression and shear. Herzl[21] investigated the contact buckling and postbuckling strength of a thin rectangular plate by experimental method. Papanikolaou and Doudoumis[22] investigated the elastic behaviour of rectangular plates with unilateral contact support condition. Table 1 also shows buckling load coefficients suggested by recent researchers for shallow arch with unilateral boundary conditions.[18,23,24]

In this study, the buckling behaviour of the inner steel tube can be considered as the snap-through behaviour of a circular shallow arch because their buckled shapes were similar to each other. The buckling coefficients proposed by Kerr and Soifer[16] were adopted to estimate the buckling strength of the internal steel tube. From the study of Kerr and Soifer,[16] the buckling strength of a circular shallow arch can be calculated by equation (17). In this equation, the normalised non-dimensional pressure ($\bar{p}$) is replaced with 2·27, which is the buckling coefficient suggested by Kerr and Soifer for snap-through buckling of a circular shallow arch. Therefore, the snap-through buckling strength (f_{bk}) of the internal steel tube of an ICH RC column can be calculated by equation (18)

$$p_0 = \bar{p} \frac{EI}{R^2 t} \tag{17}$$

$$f_{\text{bk}} = 2{\cdot}27 \frac{EI}{R^2 t} \tag{18}$$

where E is the modulus of elasticity, I is the moment of inertia, R is the radius of the internal steel tube and t is the thickness of the internal steel tube.

Equation (18) can be rewritten as equation (19). To prevent the early buckling failure of the internal steel tube before the yielding of a transverse reinforcement occurs, the buckling strength of the internal steel tube must be larger than the confining pressure acting on concrete when the transverse reinforcement yields. With this design concept and equation (19), a failure criterion can be defined as equation (20). From this equation the required minimal thickness of the internal steel tube can be calculated by equation (21)

$$f_{\text{bk}} = 2{\cdot}27 \frac{EI}{R^2 t} = 2{\cdot}27 \frac{E\left(t^3/12\right)}{\left(D_i^2/4\right) t} = \frac{2{\cdot}27}{3} \frac{t^2 E}{D_i^2} \tag{19}$$

$$f_{\text{bk}} = \frac{2{\cdot}27}{3} \frac{t^2 E}{D_i^2} > f_1 = \frac{2 f_{\text{yh}} A_{\text{sp}}}{D' s} \tag{20}$$

$$t > \sqrt{\frac{6}{2{\cdot}27} \frac{D_i^2 f_{\text{yh}} A_{\text{sp}}}{D' E s}} = t_{\text{bk}} \tag{21}$$

where t_{bk} is the required minimal thickness of the internal steel tube not to be buckled.

The required minimal thickness of the internal steel tube (t_{lim}) to prevent an early failure can be defined as the larger value between t_{yt} and t_{bk} of equations (16) and (21), respectively. Therefore, the failure criteria of an ICH RC member can be redefined as equation (22) when the flat internal steel tube is used.

$$t < t_{\text{lim}} \quad \text{Failure mode 1} \tag{22a}$$

Table 1. Circular arch buckling load coefficients

Case	Buckling load coefficient				
	Kerr and Soifer[16]	Haftka *et al.*[17]	Carnoy[23]	Argyris and Symeonidis[24]	Sun and Natori[18]
Bifurcation	1·91	1·86	1·90	1·09	1·90
Snap-through	2·27	2·17	2·26	—	2·28

$$t > t_{\lim} \quad \text{Failure mode 2} \tag{22b}$$

$$t = t_{\lim} \quad \text{Failure mode 3} \tag{22c}$$

When an ICH RC member fails as failure mode 1, this failure mode can be classified into two failure modes. One is a failure by the yield of an internal tube (failure mode 1A), and the other is a failure by buckling of the internal tube (failure mode 1B). These failure modes can be determined by the comparison of the confining stress, yield strength (f_{yt}) and buckling strength (f_{bk}) of the internal tube. It is also possible to determine the failure mode with respect to the comparison of the thickness of an internal tube and its required thicknesses for yield (t_{yt}) and buckling (t_{bk}). Table 2 shows the summary of the failure criteria for the ICH RC members.

Buckling condition of corrugated internal tube

When a corrugated internal steel tube is used instead of the flat tube, the failure criteria must be modified because the buckling strength of a corrugated tube is much larger than that of a flat tube. A corrugated plate can be analysed by replacing the thickness of the plate with an equivalent thickness (Timoshenko and Woinowsky-Krieger[25]). For a corrugated sheet as shown in Fig. 11, the flexural rigidity of a plate for each direction of x- and y-axes is given as equations (23) and (24) when the corrugation is described as a sine function

$$D_x = \frac{l}{S}\frac{Eh^3}{12(1-\nu^2)} \tag{23}$$

$$D_y = EI \tag{24}$$

where l is the length of one-half a wave, S is the arc-length of one-half a wave and ν is Poisson's ratio of a corrugated sheet or plate.

The arc-length of one-half a wave (S) and the moment of inertia (I) can be approximately calculated by equation (25) and equation (26) using height of a wave (f) presented by Timoshenko and Woinowsky-Krieger[25]

$$S = l\left(1 + \frac{\pi^2 f^2}{4l^2}\right) \tag{25}$$

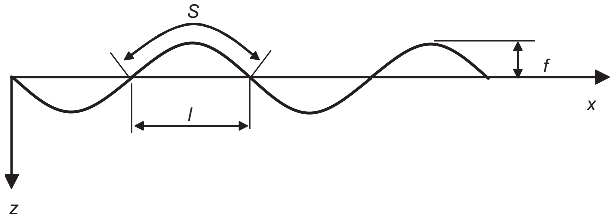

Fig. 11. Corrugated sheet

$$I = \frac{f^2 h}{2}\left[1 - \frac{0{\cdot}81}{1 + 2{\cdot}5(f/2l)^2}\right] \tag{26}$$

If t_{eq} is an equivalent thickness of the corrugated tube, equations (27) and (28) are given when the thickness of the corrugate plate (h) is replaced by the thickness of the tube (t)

$$I = \frac{f^2 t}{2}\left[1 - \frac{0{\cdot}81}{1 + 2{\cdot}5(f/2l)^2}\right] = \frac{t^3_{eq}}{12} \tag{27}$$

$$t_{eq} = \sqrt[3]{6f^2 t\left[1 - \frac{0{\cdot}81}{1 + 2{\cdot}5(f/2l)^2}\right]} \tag{28}$$

Summary of application to concrete model in ICH RC column member

The concrete in an ICH RC member is confined triaxially until the failure of the internal tube, and the concrete is confined biaxially after the failure of the internal tube. Before the internal tube fails, the confining stress and the maximum strength of concrete are given as equations (13) and (2), respectively. After the failure of the internal tube, the effective confining stress is given equation (7). The maximum strength of concrete under biaxial confinement is given as equation (8). Thus the stress–strain relation path is changed at the point of the failure of the internal tube (irregular point). The stress–strain relation is given as equation (1) and the strain at peak strength of confined concrete (ε_{cc}) is given as equation (29) suggested by Mander *et al.*[14]

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f'_{cc}}{f'_c} - 1\right)\right] \tag{29}$$

Table 2. Failure criteria of ICH RC members

		Stress criteria	Thickness criteria
Failure mode 1	A	$f_{\lim} < f_{tube}$ and $f_{yt} < f_{bk}$	$t_{\lim} > t$ and $t_{yt} > t_{bk}$
	B	$f_{\lim} < f_{tube}$ and $f_{yt} > f_{bk}$	$t_{\lim} > t$ and $t_{yt} < t_{bk}$
Failure mode 2		$f_{\lim} > f_{tube}$	$t_{\lim} < t$
Failure mode 3		$f_{\lim} = f_{tube}$	$t_{\lim} = t$

Note: $f_{\lim}$ = smaller (f_{yt}, f_{bk}), $t_{\lim}$ = larger (t_{yt}, t_{bk})

A useful limitation of compressive strain of concrete is when the fracture of transverse confining steel initiates. It may be estimated by equating the strain-energy capacity of the transverse steel as it is strained to peak stress (f_{uh}) to the increase in energy absorbed by the concrete, resulting from confinement. This increase in absorbed energy is the area under the stress–strain curve of the confined concrete in Fig. 1. A conservative estimate for ultimate compression strain is given as equation (30) suggested by Mander *et al.*[14] This has been formulated from considerations of confined sections under axial compression when used to estimate ultimate compression strain of sections subjected to bending and axial compression.

$$\varepsilon_{cu} = 0{\cdot}004 + \frac{1{\cdot}4\rho_s f_{yh}\varepsilon_{su}}{f'_{co}} \tag{30}$$

where ε_{su} is steel strain at maximum tensile stress and ρ_s is the volumetric ratio of confining steel.

Numerical evaluation and parametric study

A FORTRAN computer program was developed using the new concrete model for an ICH RC column. Fig. 12 shows the analysis procedure for the stress–strain relationship of the concrete in an ICH RC member. With the developed analysis program, some example models were analysed for ICH RC members that have flat internal steel tubes (ICH RC-FT) and have corrugated internal steel tubes (ICH RC-CT). The thickness of the internal steel tube and hollow ratio were chosen as the mainly affecting parameter on the strength of the concrete.

Figure 13 shows the sectional dimensions of the example models for ICH RC-FT and ICH RC-CT members. All transverse reinforcements have the diameter of 1·3 cm and the spacing between the adjacent two transverse reinforcements is 10 cm. The strength of unconfined concrete was set as 25 MPa. The yield strength and ultimate strain of the transverse reinforcement were set as 294 MPa and 0·20 respectively. For these example models, the internal tubes were assumed to be made of steel. The yield strength of the internal tube was set as 196 MPa and its modulus of elasticity was set as 206 GPa. The thickness of the internal tube was set in the range of zero to 5 mm ($t = 0$, 1, 2, 3, 4, 5 mm). For the corrugated tube, it is assumed that the corrugation wave has a length of 5 cm and a height of 2 cm.

Figure 14 shows the stress–strain relations of ICH RC-FT members with their hollow diameters of 140 cm. In this case, to prevent yielding and buckling, the required minimal thicknesses are 1·39 mm and 2·21 mm respectively. Thus, when the thickness of the internal tube is less than 3 mm in the example models, the internal tube is buckled. Unless the thickness of the internal tube is less than 3 mm, the ICH RC-FT member has almost same stress–strain path as that of a solid RC member because the triaxial confinement is maintained until the fracture of the transverse reinforcement. As shown in Fig. 14, ICH RC-FT members provide higher strength and ductility than the hollow RC member. Fig. 15 shows an irregular point on the stress–strain path of ICH RC-FT when its internal tube has the thickness of 2 mm. Because the failures of the members depend on the yield of their transverse reinforcements when their internal tube have the thicknesses of 3, 4 and 5 mm, the stress–strain paths are overlapped.

Figure 16 shows the stress–strain relations for ICH RC-FT members when their hollow sections have diameters of 150 cm. The internal tubes of thickness 3, 4 and 5 mm do not fail until the fracture of the outer transverse reinforcements. When the internal tube has a thickness of 1 or 2 mm, the ICH RC-FT member fails by the buckling failure of the internal tube. The required minimal thicknesses of the internal tube not to yield and not to be buckled are calculated as 1·49 and 2·37 mm, respectively.

Figure 17 shows the stress–strain relations for ICH RC-FT members with hollow sections of diameter 160 cm. In order for the internal tube not to yield or become buckled, the required thicknesses are calculated as 1·59 and 2·53 mm, respectively. The concrete of the member with the internal tube of thickness 3, 4 or 5 mm is completely confined until the outer transverse reinforcement yields. When an internal tube fails early, the failure mode depends on the thickness–diameter ratio of the internal tube. If an internal tube has a large thickness-diameter ratio, the tube fails by yielding condition. If an internal tube has a small thickness–diameter ratio, the tube fails by buckling. As the thickness–diameter ratio of the internal tube controls the failure mode of an ICH RC member, the design criteria must be focused on the thickness–diameter ratio of the internal tube.

Figure 18 shows the stress–strain relations for ICH RC-CT members. Fig. 19 shows an irregular point on the stress–strain curve when the thickness of the internal tube is 1 mm. This irregular point results from the failure of the internal tube. Because of the high buckling strength of the corrugated tube, the internal corrugated tube failed by yielding. For the ICH RC-CT members with a hollow ratio of 0·7, the minimal thicknesses of the internal tube that are required to prevent yielding and buckling are 1·39 and 0·53 mm, respectively. For the ICH RC-FT member and the ICH RC-CT member which have a thickness of 2 mm and a hollow ratio of 0·7, the flat tube is buckled but the corrugated tube does not fail by buckling.

Figures 20 and 21 show the stress–strain relations for ICH RC-CT members when their hollow diameters are 150 and 160 cm, respectively. For the members in Fig. 20, the required minimal thicknesses of the internal tube are 1·49 mm for yielding and 0·65 mm for

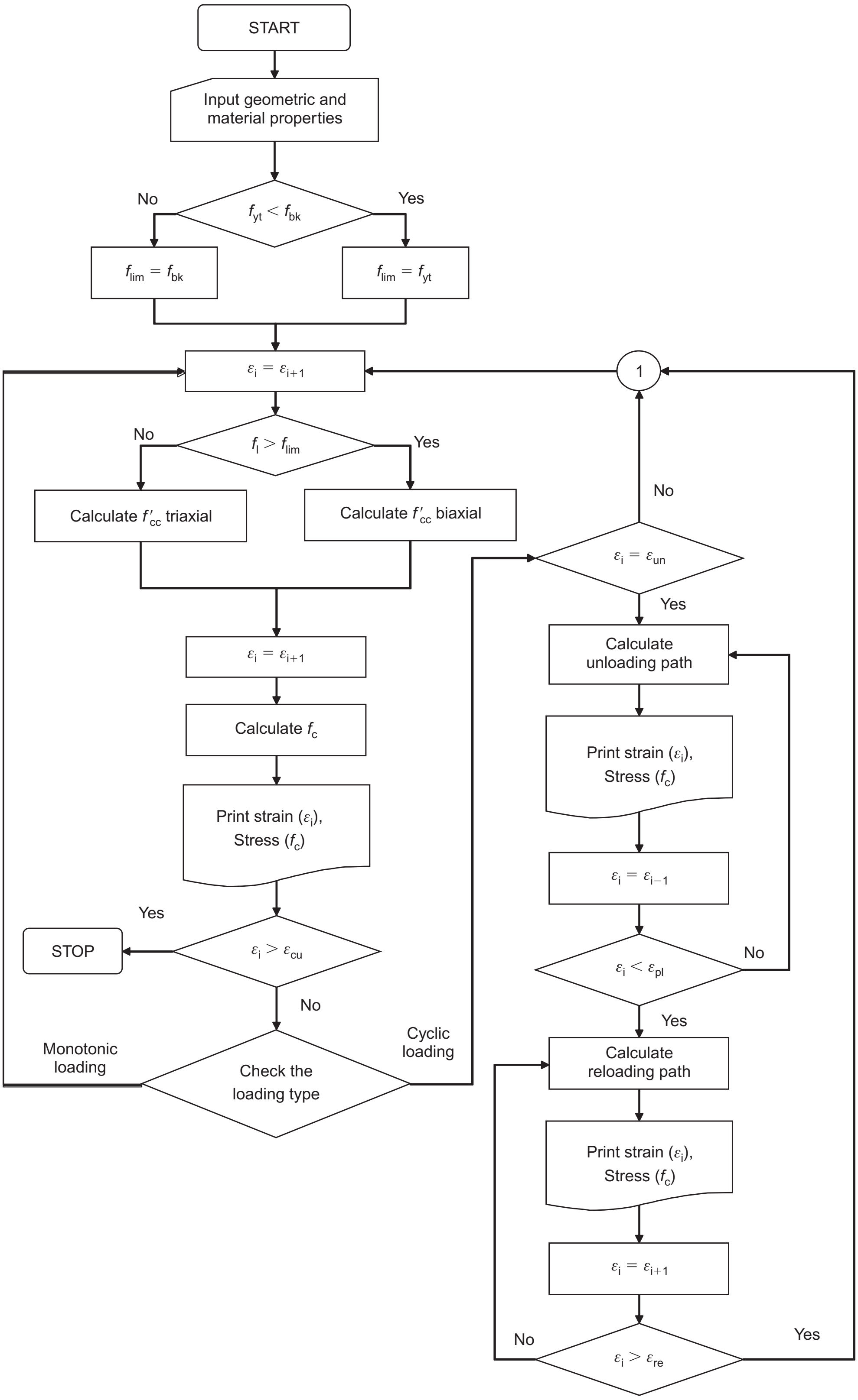

Fig. 12. Calculation procedure for proposed stress–strain model of concrete in ICH RC Column

buckling. For the members in Fig. 21, the required minimal thicknesses of the tube are 1·59 and 0·79 mm to avoid yielding and buckling failure, respectively.

Summary and concluding remarks

A non-linear concrete model for a new-type ICH RC column was developed, bearing in mind the confining

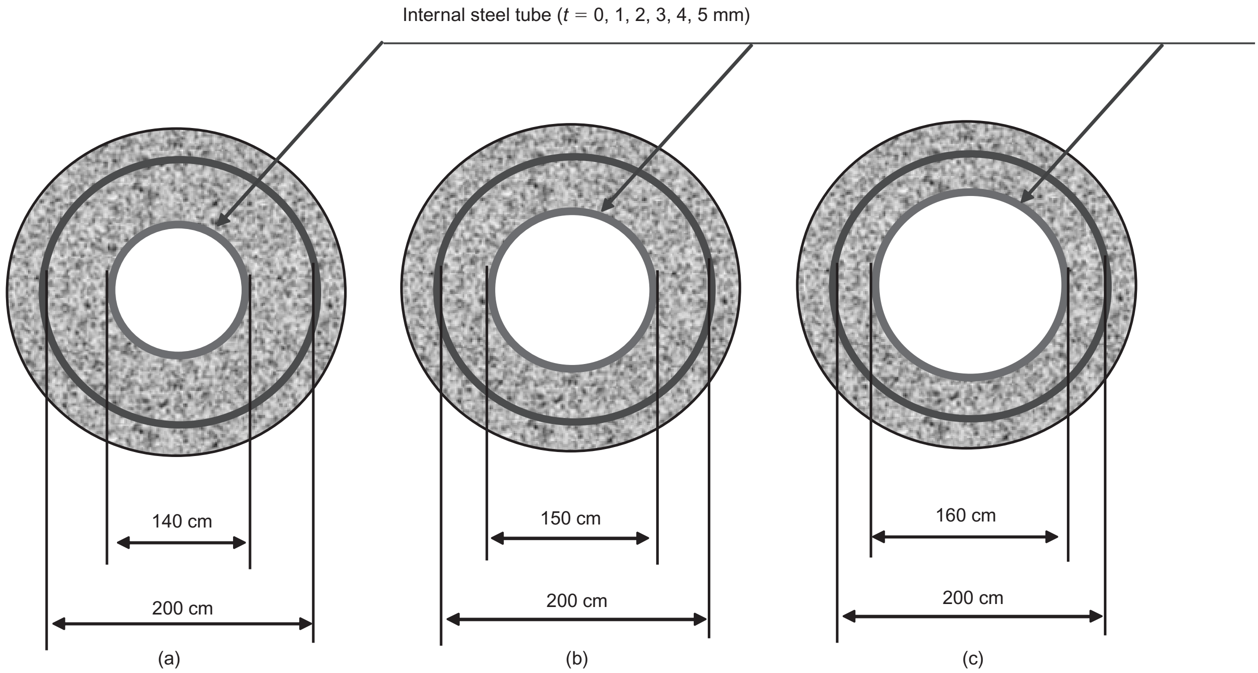

Fig. 13. Cross-sectional dimensions of example models

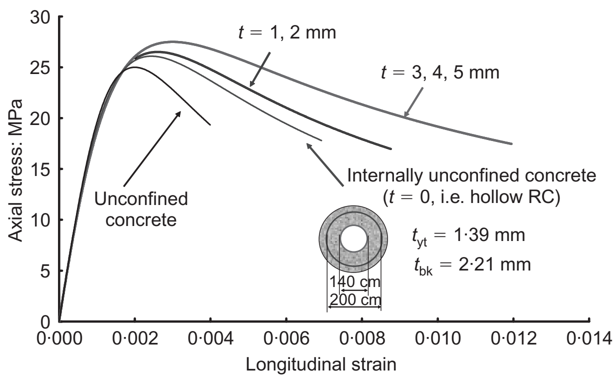

Fig. 14. Stress–strain relations of confined concrete in ICH RC-FT members (hollow ratio = 0·7)

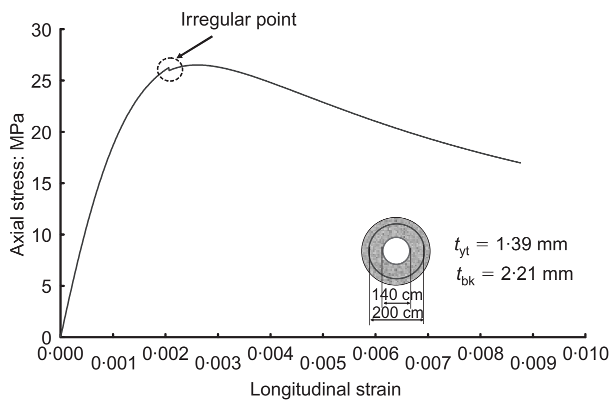

Fig. 15. Stress–strain relation of confined concrete in ICH RC-FT member (hollow ratio = 0·7, t = 2 mm)

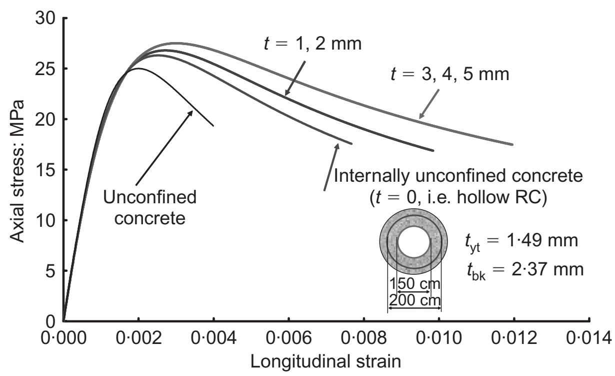

Fig. 16. Stress–strain relations of confined concrete in ICH RC-FT members (hollow ratio = 0·75)

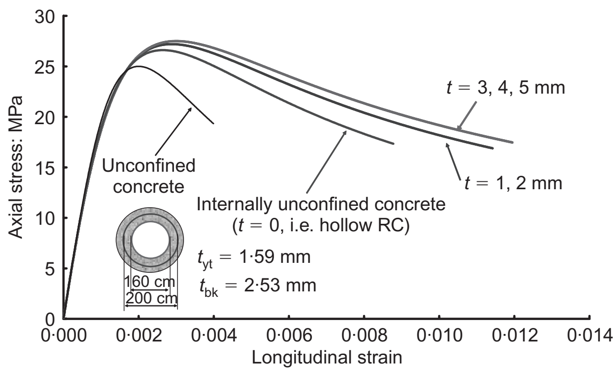

Fig. 17. Stress–strain relations of confined concrete in ICH RC-FT members (hollow ratio = 0·8)

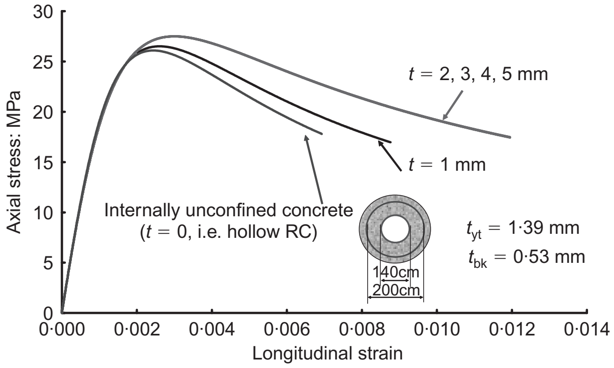

Fig. 18 Stress–strain relations of confined concrete in ICH RC-CT members (hollow ratio = 0·7)

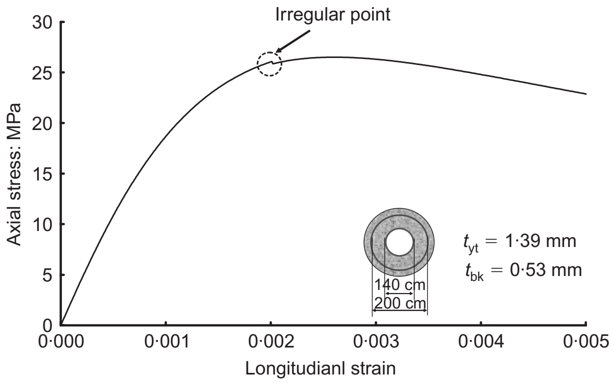

Fig. 19. Stress–strain relation of confined concrete in an ICH RC-CT member (hollow ratio = 0·7, t = 1 mm)

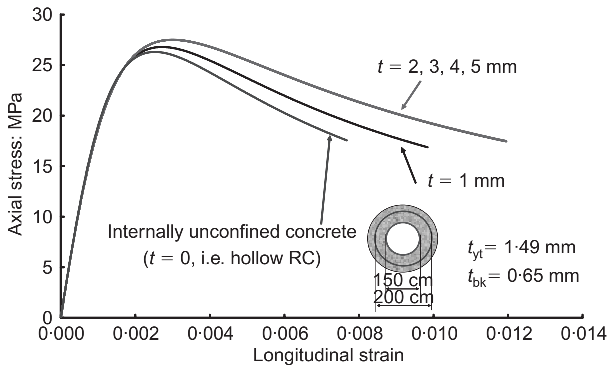

Fig. 20. Stress–strain relations of confined concrete in ICH RC-CT members (hollow ratio = 0·75)

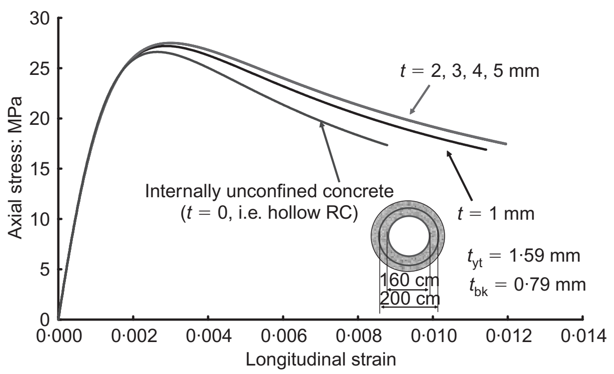

Fig. 21. Stress–strain relations of confined concrete in ICH RC-CT members (hollow ratio = 0·8)

effects of concrete. The developed concrete model was based on the study of the uniaxial stress–strain model of Mander *et al*.[14] and extended for the new-type ICH RC column using the definition of confining pressure according to several failure modes. When an internal tube does not fail before the yield failure of an outer transverse reinforcement, the concrete in the ICH RC column is confined triaxially, similar to that in a solid RC column. When an internal tube fails by buckling or yielding before the failure of the outer transverse reinforcement, the concrete in the ICH RC column is confined biaxially, similarly to that in a hollow RC column. To reduce material and to maximise buckling strength of the internal tube, a corrugated tube can be settled inside an ICH RC column. For the ICH RC column with the corrugated tube, the yield strength of the tube controls the failure of the internal tube in most cases.

The strength and ductility of concrete in an ICH RC column can be controlled by the thickness of the internal tube as well as the transverse reinforcement ratio. The thickness should satisfy the minimal requirement for the safety of the column. If the thickness of the internal tube is larger than the minimal required thickness, the concrete in the ICH column is triaxially confined and shows significantly enhanced strength and ductility. For the economic and optimised confinement of the concrete, the required minimal thickness (t_{lim}) will satisfy the design of the internal tube. Regarding column behaviour, the thickness of the internal tube will affect the column behaviour because the internal tube will contribute to the flexural rigidity of the column. Thus, the development of an ICH RC column model based on the concrete model in this study and parametric studies is necessary for the optimised design of an ICH RC column.

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